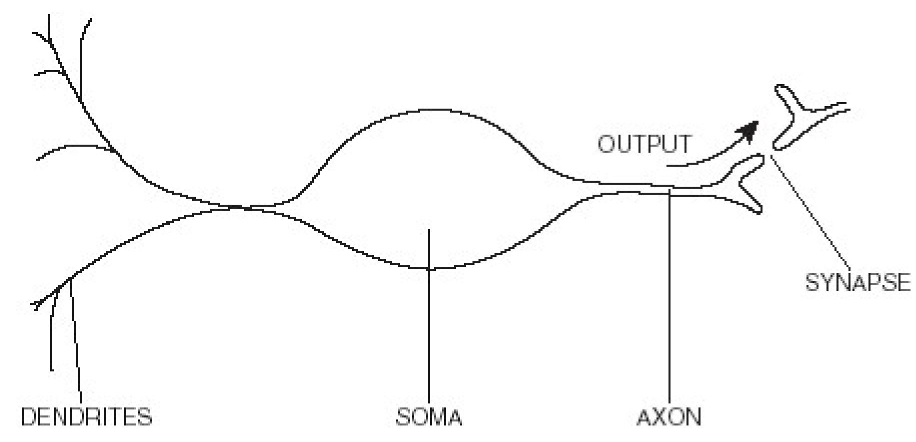
Neural Networks

Artificial Neural Networks

- Artificial neural networks (ANNs) provide a practical method for learning
  - real-valued functions
  - discrete-valued functions
  - vector-valued functions
- Robust to errors in training data
- Successfully applied to such problems as
  - interpreting visual scenes
  - speech recognition
  - learning robot control strategies

Biological Neurons

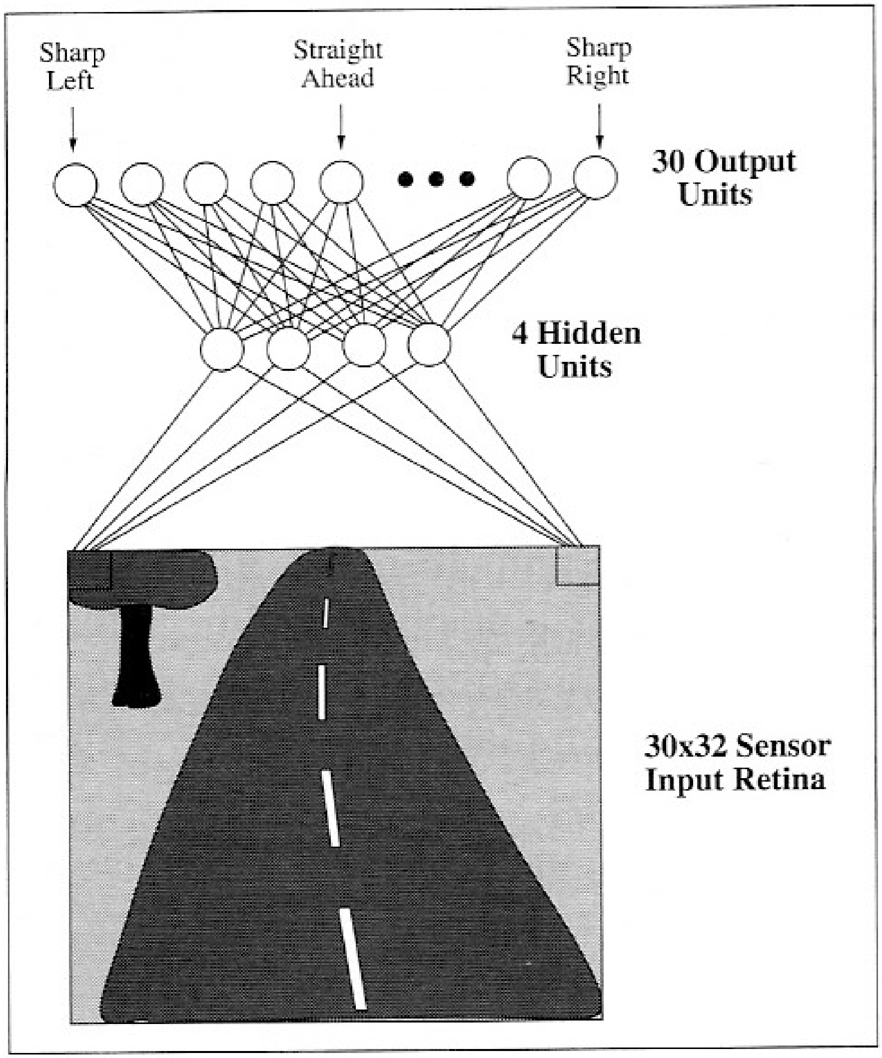
- The human brain is made up of billions of simple processing units – neurons.

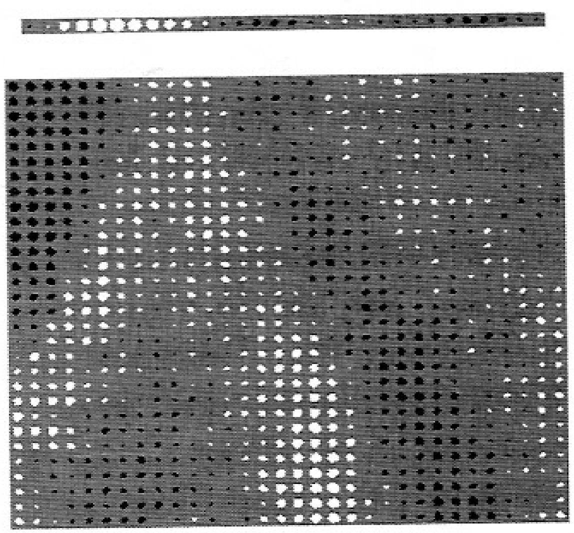


- Inputs are received on dendrites, and if the input levels are over a threshold, the neuron fires, passing a signal through the axon to the synapse which then connects to another neuron.

Neural Network Representation

- ALVINN uses a learned ANN to steer an autonomous vehicle driving at normal speeds on public highways
  - Input to network: 30x32 grid of pixel intensities obtained from a forward-pointed camera mounted on the vehicle
  - Output: direction in which the vehicle is steered
  - Trained to mimic observed steering commands of a human driving the vehicle for approximately 5 minutes


Sharp
Left
Straight
Ahead
Sharp
Right
30 Output
Units
4 Hidden
Units
30x32 Sensor
Input Retina



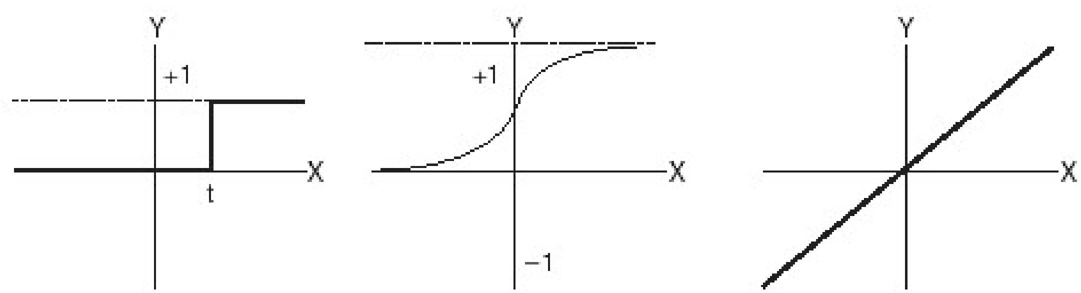
Appropriate problems

- ANN learning well-suit to problems which the training data corresponds to noisy, complex data (inputs from cameras or microphones)
- Can also be used for problems with symbolic representations
- Most appropriate for problems where
  - Instances have many attribute-value pairs
  - Target function output may be discrete-valued, real-valued, or a vector of several real- or discrete-valued attributes
  - Training examples may contain errors
  - Long training times are acceptable
  - Fast evaluation of the learned target function may be required
  - The ability for humans to understand the learned target function is not important

Artificial Neurons (1)

- Artificial neurons are based on biological neurons.
- Each neuron in the network receives one or more inputs.
- An activation function is applied to the inputs, which determines the output of the neuron – the activation level.

- The charts on the right show three typical activation functions.



(a) Step funtion (b) Sigmoid funtion (c) Linear funtion

Artificial Neurons (2)

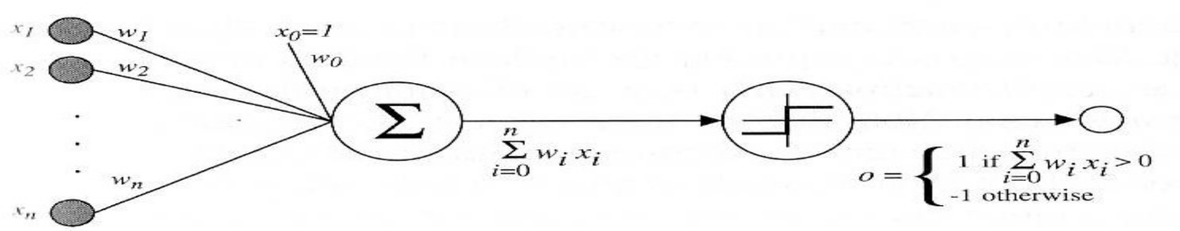
- A typical activation function works as follows:

$$X = \sum_{i=1}^{n} w_i x_i \qquad\qquad Y = \begin{cases} +1 & for\ X > t \\ 0 & for\ X \le t \end{cases}$$

- Each node *i* has a weight, w_i associated with it. The input to node *i* is x_i.
- *t* is the threshold.
- So if the weighted sum of the inputs to the neuron is above the threshold, then the neuron fires.

Perceptrons

- A perceptron is a single neuron that classifies a set of inputs into one of two categories (usually 1 or -1).
- If the inputs are in the form of a grid, a perceptron can be used to recognize visual images of shapes.
- The perceptron usually uses a step function, which returns 1 if the weighted sum of inputs exceeds a threshold, and 0 otherwise.

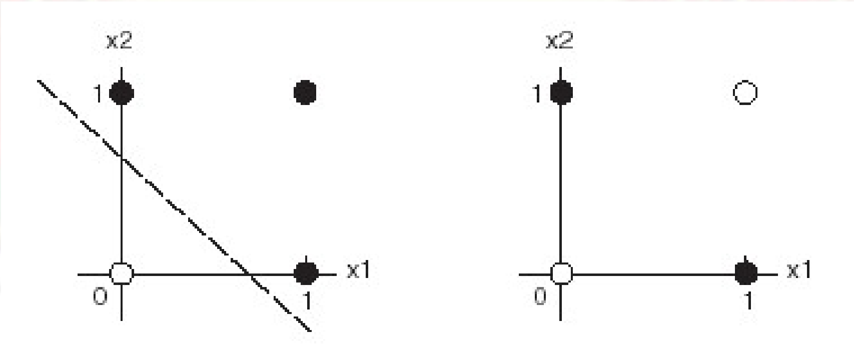


Training Perceptrons

- Learning involves choosing values for the weights
- The perceptron is trained as follows:
  - First, inputs are given random weights (usually between –0.5 and 0.5).
  - An item of training data is presented. If the perceptron mis-classifies it, the weights are modified according to the following: $$w_i \leftarrow w_i + \left(a \times x_i \times (t - o)\right)$$
    - where *t* is the target output for the training example, *o* is the output generated by the preceptron and *a* is the learning rate, between 0 and 1 (usually small such as 0.1)
- Cycle through training examples until successfully classify all examples
  - Each cycle known as an **epoch**

Bias of Perceptrons

- Perceptrons can only classify linearly separable functions.
- The first of the following graphs shows a linearly separable function (OR).
- The second is not linearly separable (Exclusive-OR).

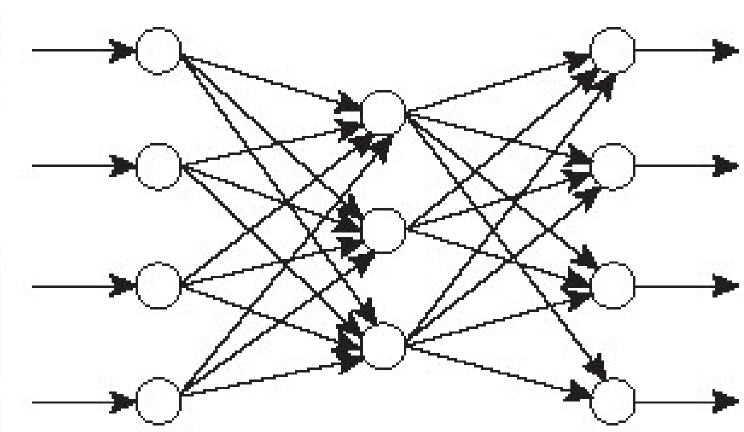


Convergence

- Perceptron training rule only converges when training examples are linearly separable and a has a small learning constant
- Another approach uses the *delta rule* and gradient descent
  - Same basic rule for finding update value
  - Changes
    - Do not incorporate the threshold in the output value (unthresholded perceptron)
    - Wait to update weight until cycle is complete
  - Converges asymptotically toward the minimum error hypothesis, possibly requiring unbounded time, but converges regardless of whether the training data are linearly separable

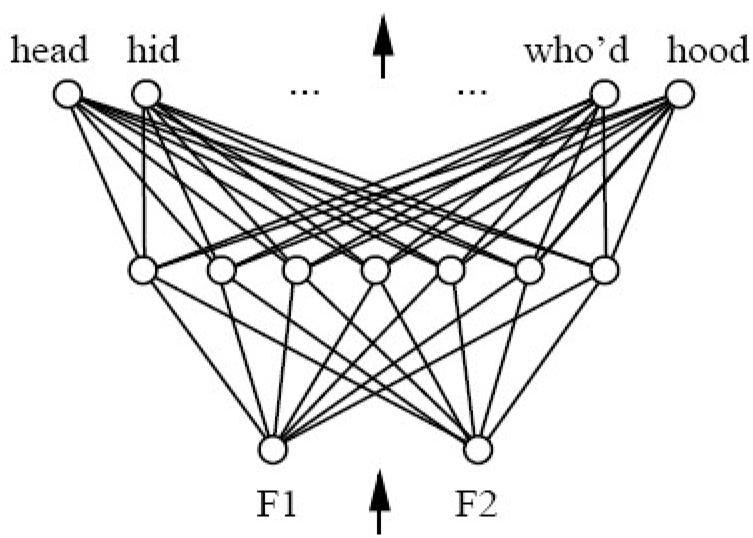
Multilayer Neural Networks

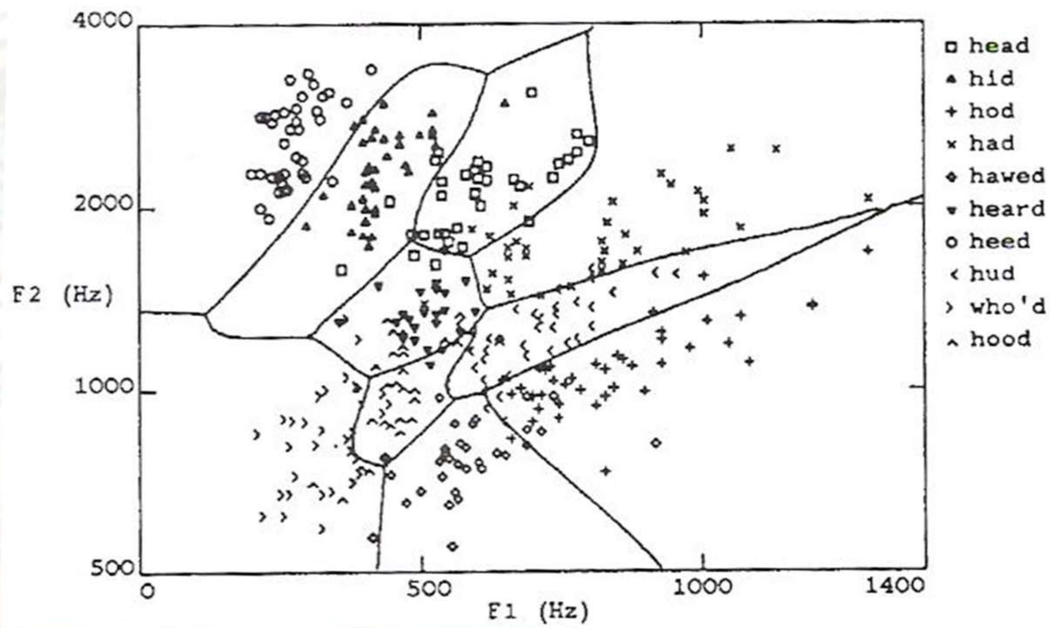
- Multilayer neural networks can classify a range of functions, including non linearly separable ones.
- Each input layer neuron connects to all neurons in the hidden layer.
- The neurons in the hidden layer connect to all neurons in the output layer.



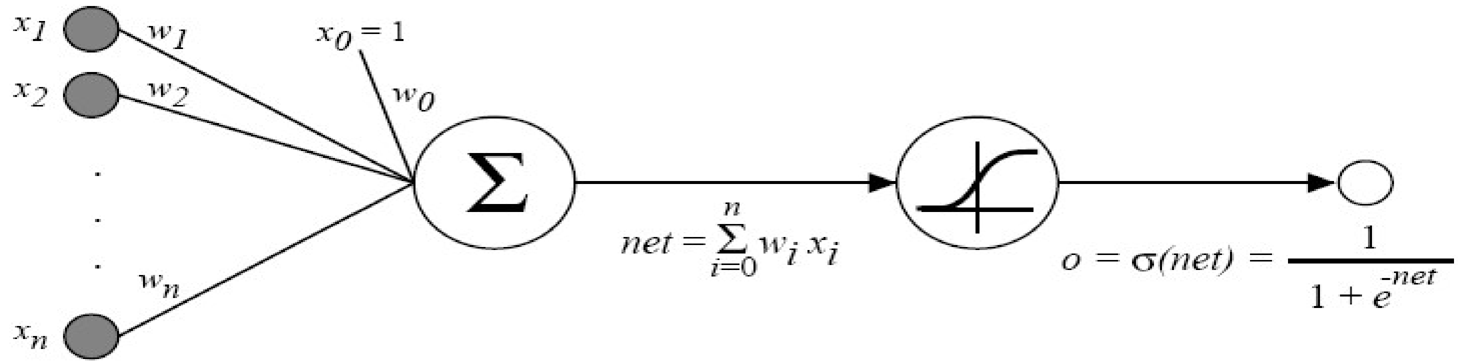
A feed-forward network

Speech Recognition ANN





Sigmoid Unit



- $\sigma(x)$ is the sigmoid function $\frac{1}{1+e^{-x}}$
- Nice property: differentiable $\frac{\partial \sigma(x)}{\partial x} = \sigma(x)(1-\sigma(x))$
- Derive gradient descent rules to train
  - One sigmoid unit - node
  - Multilayer networks of sigmoid units

Backpropagation

- Multilayer neural networks learn in the same way as perceptrons.
- However, there are many more weights, and it is important to assign credit (or blame) correctly when changing weights.
- *E* sums the errors over all of the network output units

$$E(\vec{w}) \equiv \frac{1}{2} \sum_{d \in D} \sum_{k \in outputs} (t_{kd} - o_{kd})^2$$

Backpropagation Algorithm

- Create a feed-forward network with n_{in} inputs, n_{hidden} hidden units, and n_{out} output units.
- Initialize all network weights to small random numbers
- Until termination condition is met, Do
  - For each <*x*,*t*> in training examples, Do

    Propagate the input forward through the network:

    1. Input the instance x to the network and compute the output o_u of every unit u in the network

    Propagate the errors backward through the network:

    2. For each network output unit k, calculate its error term δ_k

    $$\delta_k \leftarrow o_k(1-o_k)(t_k-o_k)$$

    3. For each hidden unit h, calculate its error term δ_h

    $$\delta_h \leftarrow o_h(1-o_h) \sum_{k \in outputs} w_{kh}\delta_k$$

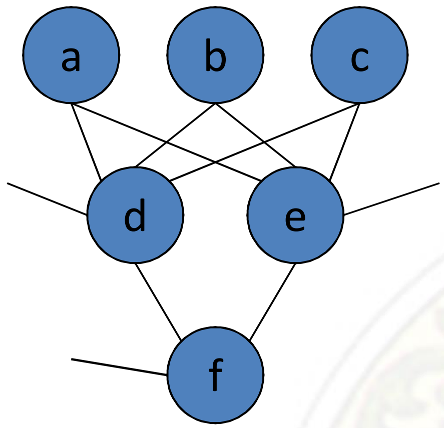
    4. Update each network weight w_{ji}

    $$w_{ji} \leftarrow w_{ji} + \Delta w_{ji}$$

    where

    $$\Delta w_{ji} = \alpha \delta_j x_{ji}$$

Example: Learning AND



Initial Weights:
w_da = .2
w_db = .1
w_dc = -.1
w_d0 = .1

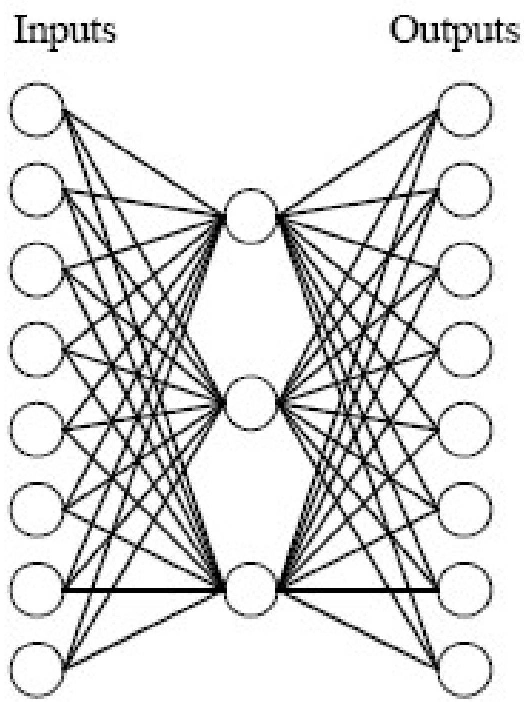
w_ea = -.5
w_eb = .3
w_ec = -.2
w_e0 = 0

w_fd = .4
w_fe = -.2
w_f0 = -.1

Training Data:
AND(1,0,1) = 0
AND(1,1,1) = 1

Alpha = 0.1

Hidden Layer representation



Target Function:

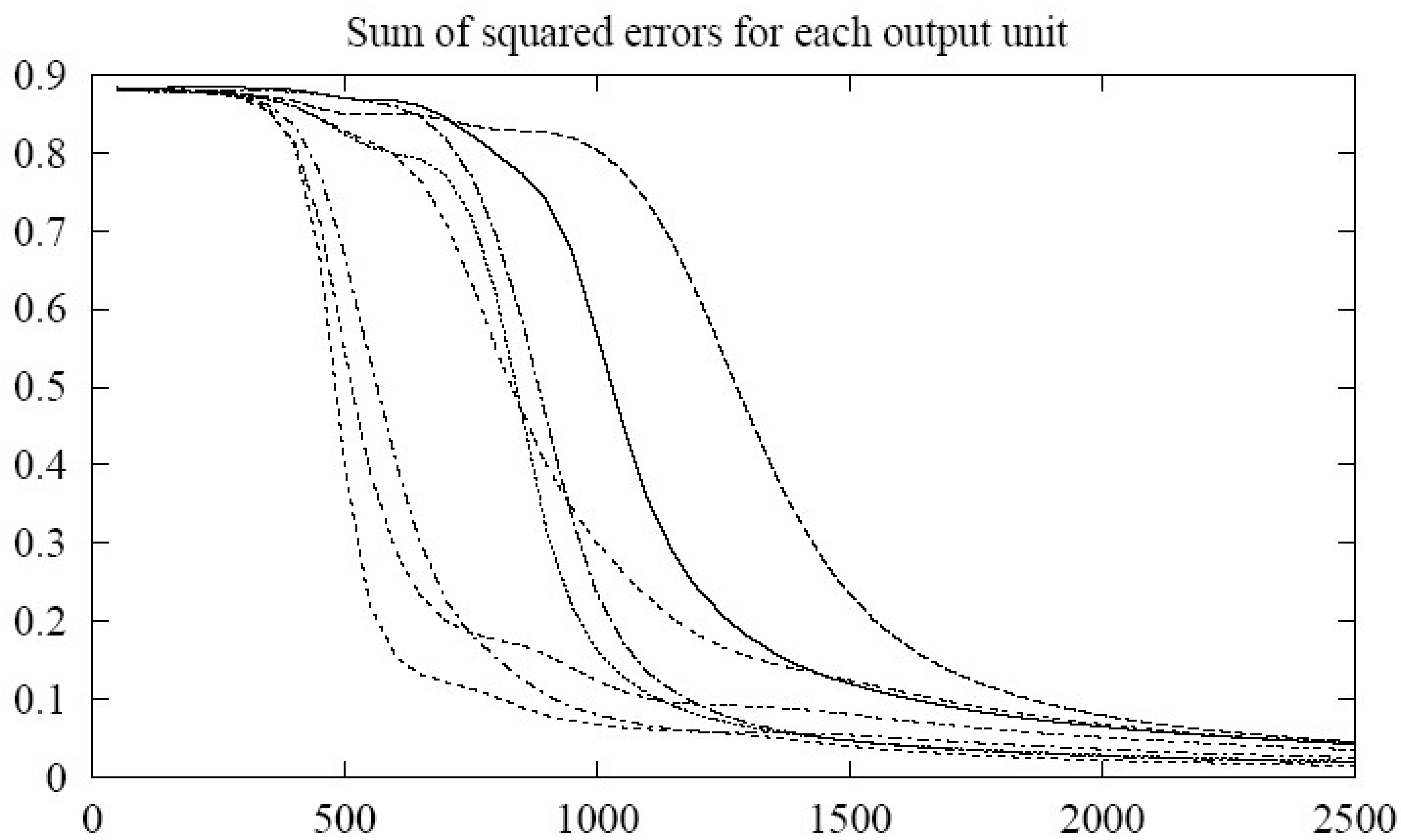
Input		Output
10000000	→	10000000
01000000	→	01000000
00100000	→	00100000
00010000	→	00010000
00001000	→	00001000
00000100	→	00000100
00000010	→	00000010
00000001	→	00000001

Can this be learned?

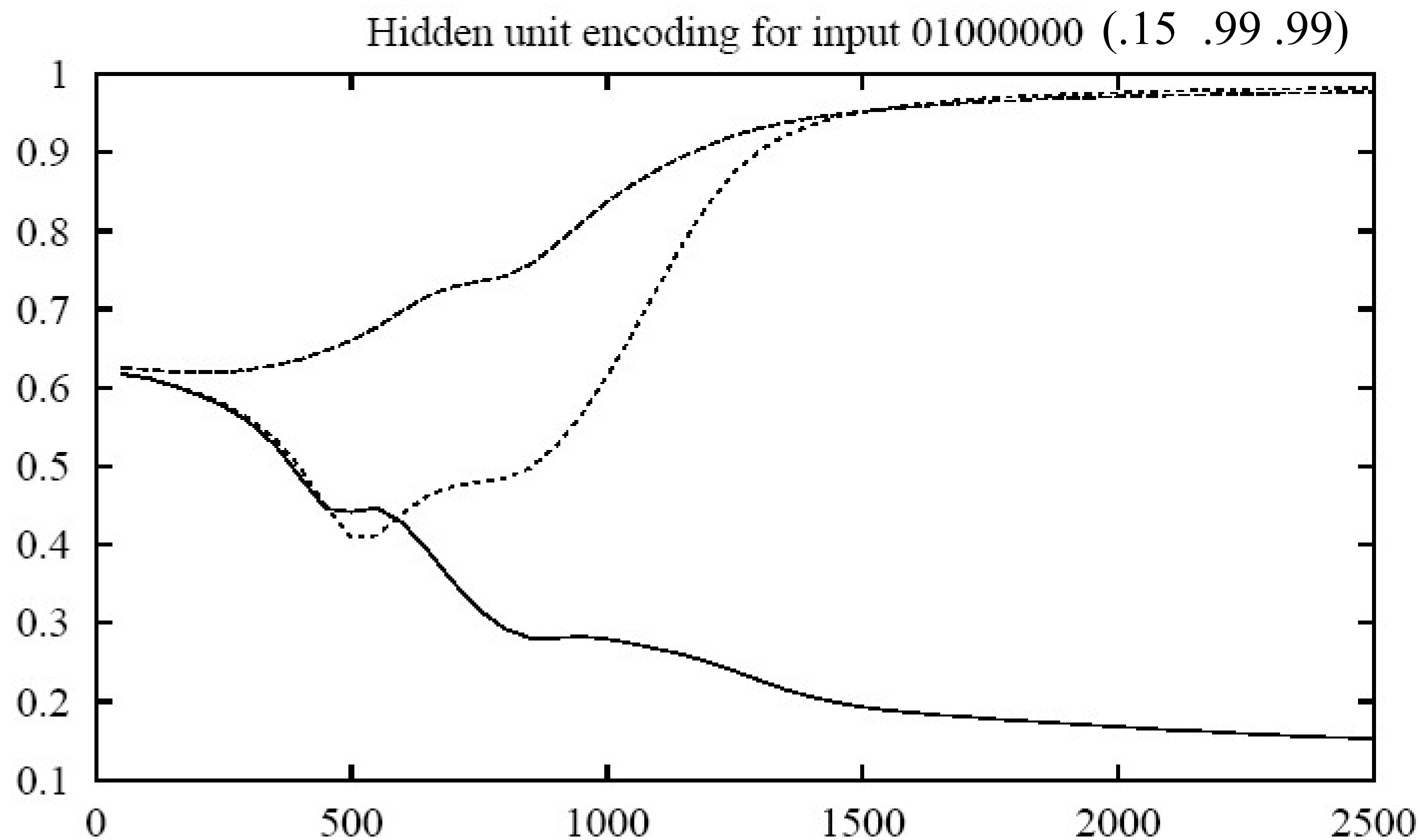
Yes

Input		Hidden Values		Output
10000000	→	.89 .04 .08	→	10000000
01000000	→	.15 .99 .99	→	01000000
00100000	→	.01 .97 .27	→	00100000
00010000	→	.99 .97 .71	→	00010000
00001000	→	.03 .05 .02	→	00001000
00000100	→	.01 .11 .88	→	00000100
00000010	→	.80 .01 .98	→	00000010
00000001	→	.60 .94 .01	→	00000001

Plots of Squared Error



Hidden Unit


Hidden unit encoding for input 01000000 (.15 .99 .99)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
500
1000
1500
2000
2500

Evolving weights

Momentum

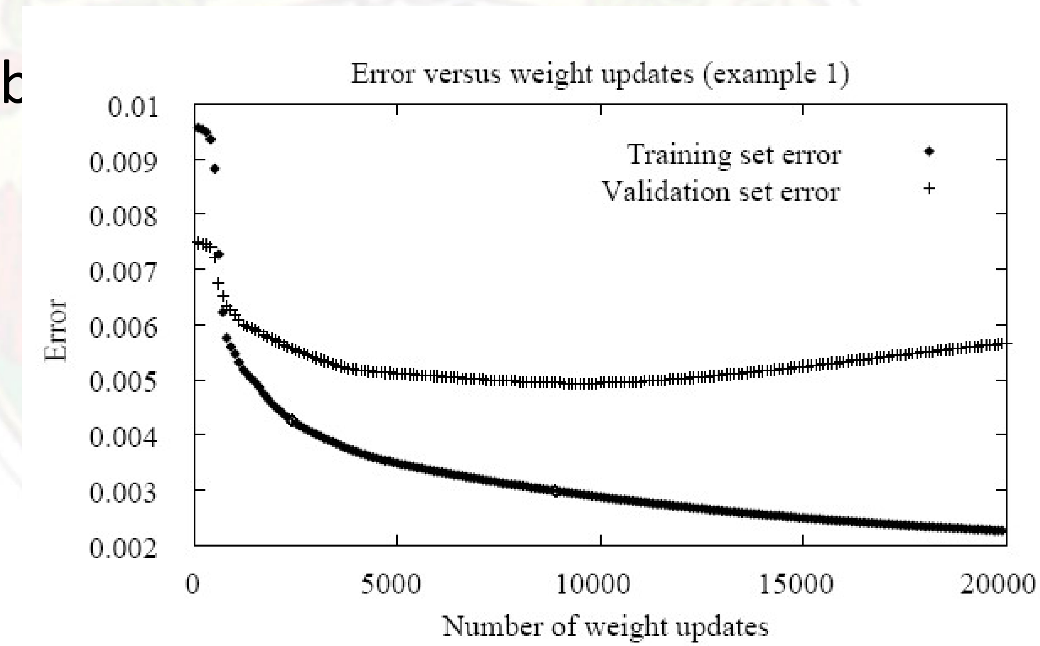
- One of many variations
- Modify the update rule by making the weight update on the *n*th iteration depend partially on the update that occurred in the (*n*-1)th iteration

$$\Delta w_{ji}(n) = \alpha \delta_j x_{ji} + \beta \Delta w_{ji}(n-1)$$

- Minimizes error over training examples
- Speeds up training since it can take 1000s of iterations

When to stop training

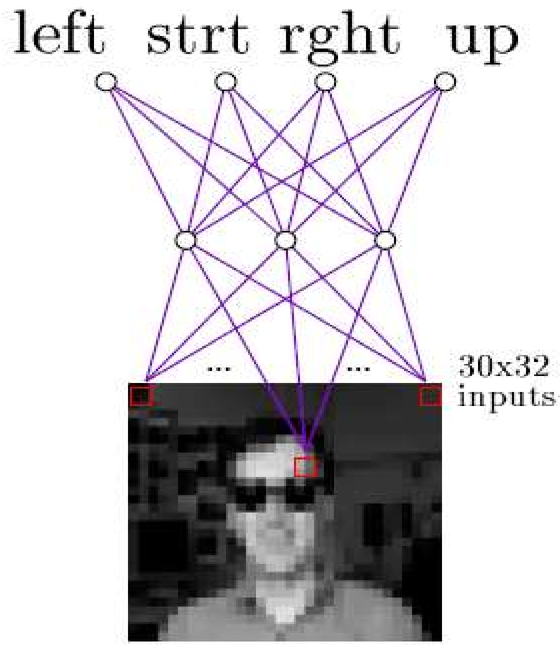
- Continue until error falls below some predefined threshold
  - Bad choice because Backpropagation is susceptible to overfitting
  - Won't be ab … a



Cross Validation

- Common approach to avoid overfitting
- Reserve part of the training data for testing
- m examples are partitioned into k disjoint subsets
- Run the procedure k times
  - Each time a different one of these subsets is used as validation
- Determine the number of iterations that yield the best performance
- Mean of the number of iterations is used to train all n examples

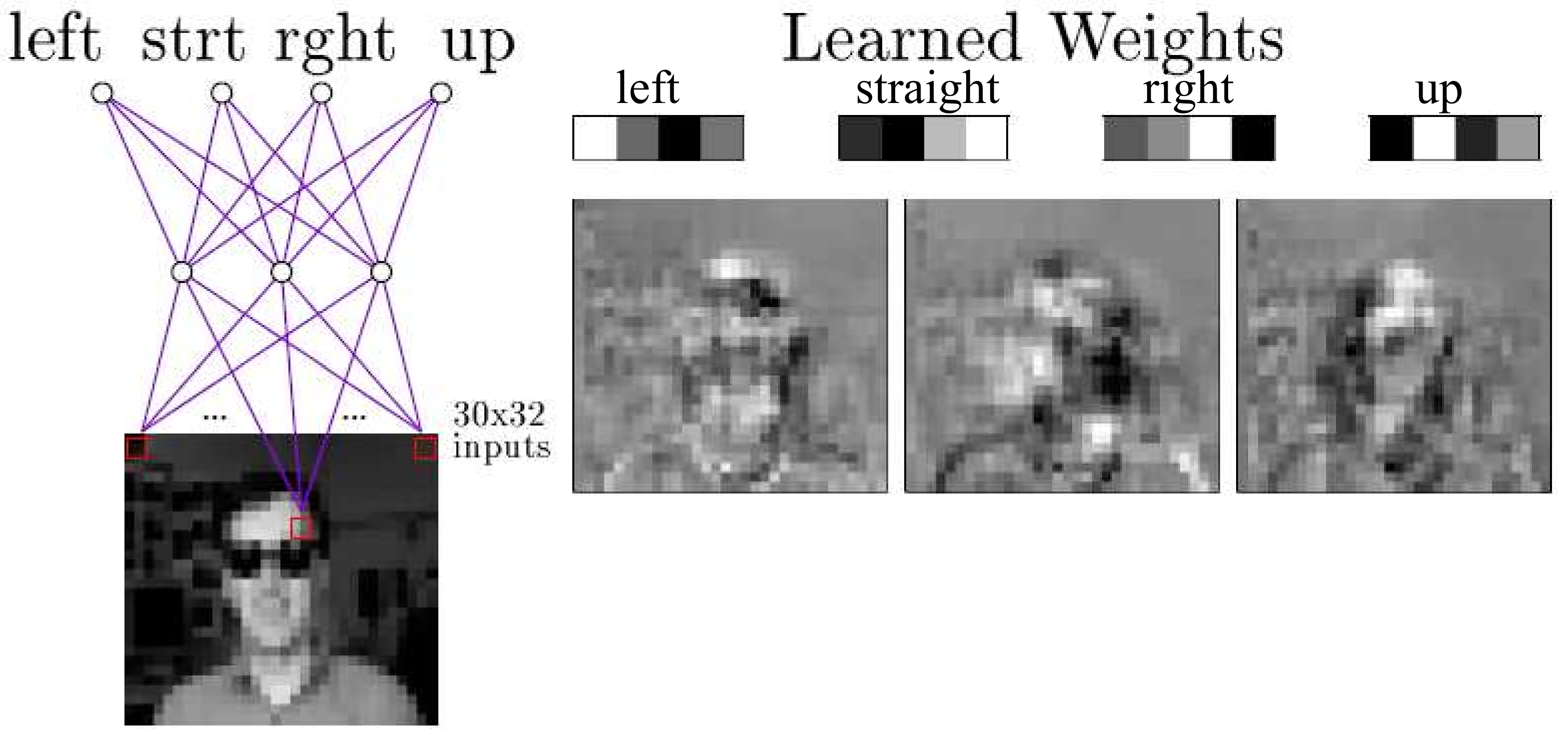
Neural Nets for Face Recognition



Typical input images

C

Hidden Unit Weights



Error gradient for the sigmoid function

$$
\begin{aligned}
\frac{\partial E}{\partial w_i} &= \frac{\partial}{\partial w_i} \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2 \\
&= \frac{1}{2} \sum_d \frac{\partial}{\partial w_i} (t_d - o_d)^2 \\
&= \frac{1}{2} \sum_d 2(t_d - o_d) \frac{\partial}{\partial w_i} (t_d - o_d) \\
&= \sum_d (t_d - o_d) \left(- \frac{\partial o_d}{\partial w_i} \right) \\
&= - \sum_d (t_d - o_d) \frac{\partial o_d}{\partial net_d} \frac{\partial net_d}{\partial w_i}
\end{aligned}
$$

.

Error gradient for the sigmoid function

But we know:

$$\frac{\partial o_d}{\partial net_d} = \frac{\partial \sigma(net_d)}{\partial net_d} = o_d(1 - o_d)$$

$$\frac{\partial net_d}{\partial w_i} = \frac{\partial (\vec{w} \cdot \vec{x}_d)}{\partial w_i} = x_{i,d}$$

So:

$$\frac{\partial E}{\partial w_i} = -\sum_{d \in D} (t_d - o_d) o_d (1 - o_d) x_{i\,d}$$